

Black hole entropy: inside or out?

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Abstract

A trialogue. Ted, Don, and Carlo consider the nature of black hole entropy. Ted and Carlo support the idea that this entropy measures in some sense “the number of black hole microstates that can communicate with the outside world.” Don is critical of this approach, and discussion ensues, focusing on the question of whether the first law of black hole thermodynamics can be understood from a statistical mechanics point of view.

A Trialogue

The following is our rough reconstruction of a discussion that took place during the summer of 2004. The participants are Ted Jacobson, Don Marolf, and Carlo Rovelli. The topic is the interpretation of the Bekenstein-Hawking entropy S_{BH} of a black hole. The conversation takes place in three scenes, during the course of an afternoon and an evening.

Ted and Carlo support the idea that the Bekenstein-Hawking entropy measures in some sense “the number of black hole microstates that can communicate with the outside world.” Don has certain sympathies in this direction, but does not see a complete picture in which he can believe. As a result, he finds the contrasting point of view compelling. The contrasting point of view is that S_{BH} counts the total number of black hole microstates, including all configurations of the interior. As the conversation progresses, all sides attempt to learn something new about this old, but still exciting, question of black hole physics.¹

Scene I: Entanglement, outside on the terrace

The setting: A fine summer afternoon. A group of physicists stand on a terrace overlooking a restored 13th century village in the French Alpes-de-Haute-Provence.

Don: Greetings, Ted! It’s a pleasure to see you, as always. But this time particularly so: I have a question that I have been saving for you, and also

¹In the spirit of an informal discussion, citations are used only to help the reader locate arguments specifically referenced in the discussion. However, all participants implicitly draw heavily on the work of many other physicists. Complete references can be found in those few works cited below.

for Carlo, when I can catch him! I can see that this is the perfect setting to spring it upon you.

To begin, I know that you advocate the view that black hole entropy has nothing to do with the internal states of a black hole, but is rather a measure of the number of states, associated with the existence of the horizon, which can influence the outside world. That is, if one could count the complete set of microstates associated with a given black hole (and including a description of the interior), you both believe that the correct result would be far larger than the number $e^{S_{BH}}$ naturally associated with the Bekenstein-Hawking entropy.

The funny thing about your point of view is that it would remove black holes from the context of familiar statistical mechanics. This means that a number of the usual stat mech arguments would not apply directly to black holes. Yet, the original motivation for the Bekenstein-Hawking entropy is that it played *precisely* the role one would expect for the entropy of a standard statistical mechanical system. So, what I want to know is how, within your point of view, could one possibly explain the laws of black hole thermodynamics? Here I am in fact most concerned about the *first* law of black hole thermodynamics, which states that two nearby equilibrium configurations are related by

$$dE = TdS + \text{Work Terms.} \quad (1)$$

However, for completeness I'll ask you to explain the second law as well:

$$\frac{dS}{dt} \geq 0. \quad (2)$$

Ted: I don't know why you say my point of view removes black holes from the context of statistical mechanics. The outside observers do their stat. mech. calculations with all the information available to them. The states strictly behind the horizon are irrelevant for that. How this different from any other application of statistical thermodynamics?

Don: Well, in the *usual* setting, the first law is tied to the fact that if an energy dE is added to a system in equilibrium, and the system returns to equilibrium, the energy is distributed over all the available states. The entropy change dS reflects the change in the number of microstates after this re-equilibration occurs.

Ted: Exactly! This supports the argument that the states behind the horizon are irrelevant: the spacetime behind the horizon is not static, but rather quite time-dependent. Even for a static black hole, the Killing field is spacelike, not timelike in there. Everything falls towards the interior, into the singularity. There seems to be no reason to imagine that any kind of equilibration process occurs.²

Don: Exactly! I'll concede the point that the interior of a black hole does not appear to reach equilibrium in any sense we understand. But if we take this at face value, then the black hole is an *essentially* non-equilibrium system! So, why should the first law hold at all?

Ted: I think equilibrium is a question of timescales. The black hole horizon appears to static observers like a thermal state, just as in flat spacetime the Minkowski vacuum appears as a thermal state to uniformly accelerated observers. Because of the gravitational redshift, the temperature of that state goes to infinity at the horizon. An infinite temperature corresponds to infinite energies, hence infinitesimal times. Thus the equilibration time for the energy dE crossing the horizon is, in a sense, zero. And voilà, the first law emerges!

Don: Woah. Slow down a minute here.... I didn't catch that at all. First, when you say that the "equilibration time ... is ... zero," you mean that equilibrium is indeed reached, and that in fact it happens infinitely quickly?

Ted: Yes. Well, I think this is only a very good approximation, to leading order in the ratio of the Planck length to the size of the black hole. I imagine that black hole entropy is finite because some kind of Planck scale cutoff exists on the field degrees of freedom, and because the Planck length is so short compared to the other relevant scales, the entropy is dominated by Planck scale degrees of freedom, and hence it is only their equilibration that really matters. And their equilibration takes place over Planck times, in some sense.

Don: Hmmm.... I'm not sure yet how this question of timescales is relevant. So, let me put that aside for a moment. I will certainly agree that, at least in a certain sense, the exterior of a black hole reaches equilibrium.

²This point was emphasized in Rafael Sorkin's article [1] in the Chandra volume.

However, a greater concern is that the exterior of the black hole does not exist in isolation. Instead, it is coupled to the interior of black hole. And we have agreed that the interior does not reach equilibrium, so why should we be able to apply the rules of equilibrium thermodynamics to the exterior? In other words, just *how* can we conclude that the first law will emerge?

Ted: Well *that's* a fat pitch³! As our numerical relativity colleagues gratefully point out, the exterior evolution is independent of what happens inside the black hole. The “coupling” between the interior and the exterior happens, in effect, just at the horizon, again at this very short length scale where the only thing happening is the vacuum fluctuations.

Don: I don't think it's so trivial. After all, isn't it true that the *motivation* for your belief that S_{BH} does not count the total number of black hole states is that you would like the black hole to be able to absorb an infinite amount of information, so that in turn information can be “lost” inside a black hole as it evaporates?

Ted: Well, no, that's not my *motivation*, but I do believe the black hole can absorb lots of information.

Don: Then I think you're trying to have your cake, and eat it too. Such information transfer to the interior requires that we speak about the black hole horizon as being coupled to the interior.

Ted: Ah, I thought you were referring to coupling the other way; you're right of course that the exterior can affect the interior...

Don: Let me draw a Carter-Penrose diagram or two to illustrate the point. (*Don takes out a pen and paper and starts to draw Figure 1, below.*) Let's consider a spacetime which has a future horizon H . I'll draw this as a heavy line. Hmm..., I guess I'll draw the familiar asymptotically flat case, though I don't think this will be important to the story. Let me also assume that we constantly send a small stream of energy into the hole to balance the outgoing Hawking flux, so that the black hole does not evaporate.

Now, there are basically two senses in which we could attempt to discuss the physics near the black hole... where by “physics” I mean both the issue

³<http://www.enlexica.com/sp/bb/index.html>

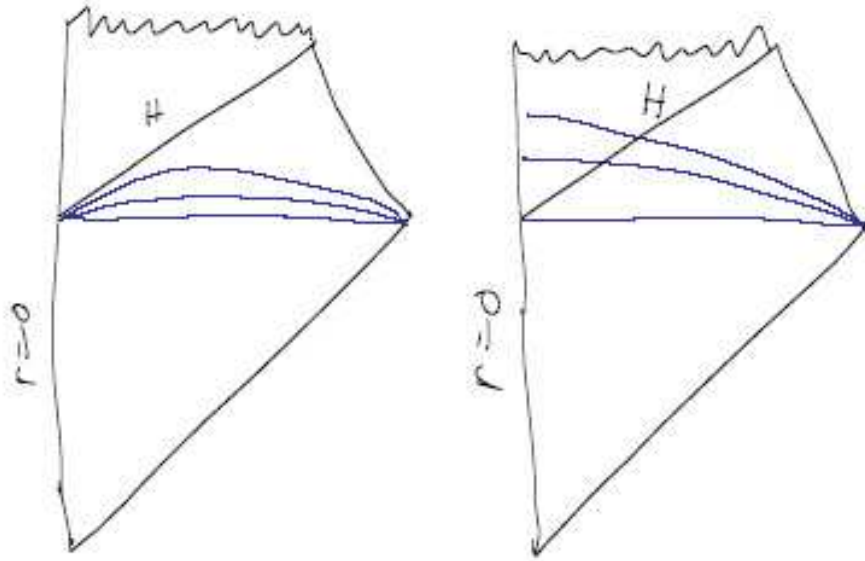


Figure 1: Don draws two Carter-Penrose diagrams, each showing a family of Cauchy surfaces.

of information loss and the approach of a black hole to equilibrium. These correspond to looking at two different families of spacelike hypersurfaces in this spacetime. This will be clearest if I make two copies of my diagram, with each copy showing one of the two families of hypersurfaces. I'll draw the hypersurfaces with dotted lines.

On the left, each hypersurface is a Cauchy surface, and each lies completely in the exterior of the black hole. I certainly agree that, so far as we know, physics in this exterior region can be described as uncoupled from the black hole interior. However, there are two reasons why I don't think this is the picture you wish to discuss. First, it is not clear in what sense the black hole "approaches equilibrium" with respect to the time evolution described by this family. Indeed, any sort of junk which falls into the black hole is present on every surface in this family and does not disappear. Instead, it moves ever closer (and ever more slowly) toward the black hole. Second, this family of hypersurfaces clearly loses no information.

In contrast, the family on the right is drawn so that each succeeding hypersurface intersects the horizon a bit farther up. I've chosen these to be

Cauchy surfaces as well, though clearly information is “lost” to the interior of the black hole. In terms of this family, I do see that the black hole approaches equilibrium after being perturbed: this is just the statement that the spacetime approaches some stationary solution as one moves up along the horizon.

However, we see that evolution in the sense described by these surfaces includes both the interior and the exterior regions. In particular, we should note that there is an exchange of energy across the horizon: positive energy falls across the horizon passing through any surface system and into the interior and, due to Hawking radiation, so does negative energy! Thus, energy is shuffled back and forth.

So, now, let me restate my original question: Classical general relativity tells us that if we actively probe a black hole, then the combined interior-surface system responds in accord with the first law of thermodynamics.

$$dE = TdS + \text{Work Terms.}$$

But how can this result be derived from your picture in which S is the entropy of only the surface system, and thus does not include the interior?

Ted: The interior is causally disconnected, hence irrelevant for the outside observer. I would assert that the entropy in the first law is the log of the number of states of the black hole that can affect the exterior.

I believe this answers the part of your question regarding how the interior could possibly be left out of consideration in deriving the first law, but it does not yet address how one could *derive* the first law, whether it be with or without the interior. I suspect you may be willing to accept my rationale for the irrelevance of the interior if I can actually account for the emergence of the first law without the interior playing any role.

Don: Now, I wouldn’t want to overly commit myself in advance... But if you can explain the first law in terms of the entropy of your “surface system” alone, then my question will indeed have been answered.

Ted: Well, the usual derivations of the first law, whether by purely classical *mechanical* calculations, or by formal semiclassical quantum gravity calculations of the thermal partition function, do not refer to the interior. But I suppose you are asking for a “statistical thermodynamics” type derivation, where the energy falling across the horizon is directly linked to the

change of the number of microstates, right? (*Don nods, smiling and raising his eyebrows.*) Hmm, lemme see here...

Well, then, I'm going to have to commit myself to a candidate for the microscopic states. I'll profess to the belief that these are the vacuum fluctuations just outside the horizon, which carry entropy since they are correlated to fluctuations inside the horizon. That is, I think the black hole entropy is vacuum entanglement entropy⁴. The fact that this scales with the horizon area follows simply from dimensional considerations. Thus there is a universal proportionality constant relating horizon area to entropy. The classical proof of the first law, together with the Hawking temperature $\hbar\kappa/2\pi$, then implies the identity of this proportionality constant as being precisely $1/4\ell_{\text{Planck}}^2$.

Don: I'm not at all sure there are enough states in such fluctuations to do the job⁵, but that's probably a side issue for our present discussion.

Ted: Hey, to me that sounds crucial for the main issue, not a side issue at all! If you're right that there are not enough states outside the horizon, then there's no way my picture could be correct. We should discuss this too!

Don: That would be great.... I would love to have a separate discussion with you sometime about all of my concerns regarding the "entanglement entropy scenario," but I think we'll have plenty to discuss today just sticking to broader questions about the first law....

Here, the main point for me is that you haven't yet gone far enough. What I am asking is how, microscopically, is the energy thermalized into your horizon degrees of freedom, in accordance with the first law.

Ted: Maybe we are making progress here, since the question has now focused down onto the equilibration process... Well, if the entropy arises from the thermal nature of the vacuum outside the horizon as viewed by uniformly accelerated observers, all I really need to explain is why the vacuum remains the vacuum at the short distance scales where the states counted by the Bekenstein-Hawking entropy reside. My answer would be that, because of decoupling, the vacuum is not excited at these short scales.

⁴Ted recommends the reference [1].

⁵Don refers to arguments in [2].

Don: I don't see how to get $dE = TdS$ from this statement. In particular, when the energy dE thermalizes it must cause *some* change, or else the entropy would not grow!

Ted: I guess my argument has been somewhat cryptic so far. Let me try to remedy that. To begin with, I think it would help to be very concrete about what you mean by deriving the first law from a microscopic description. In a typical setting of a system in thermodynamic equilibrium, the statistical description of the system is given by a density matrix ρ . Let's fix on the canonical ensemble, so $\rho = \exp(-\beta H)/Z$, where β is the inverse temperature, H is the Hamiltonian, and Z is the trace of $\exp(-\beta H)$. Suppose some energy is added to the system, but in such a way that $\delta\rho \ll \rho$. Then, as discussed in your paper⁶ with Minic and Ross, varying the entropy $S = -\text{Tr}(\rho \ln \rho)$ and the mean energy $\langle E \rangle = \text{Tr}(\rho H)$, one finds the relation

$$\delta S = \beta \delta \langle E \rangle, \quad (3)$$

hence the thermodynamic relation $dE = TdS$ that we have been calling the first law.⁷ Thus once we have a system whose density matrix is a canonical ensemble, we have a derivation of the first law, agreed?

Don: Yes, Ted, I agree...provided that you can justify the statement that your system is in a canonical ensemble, especially *after* you make the change $\delta\rho$...and provided that we are discussing the physically relevant system.

Ted: Well then here's my argument. First, I claim we know that the "surface system" is well described by a canonical ensemble, as long as it is very near the local vacuum state. Here's the reason: at short distances a little patch of the black hole horizon may be identified with a patch of a Rindler horizon in Minkowski spacetime. The Unruh effect, or what is known in algebraic quantum field theory as the Bisognano-Wichmann theorem, tells us that, as viewed from one side of a Rindler horizon, the Minkowski vacuum is a canonical ensemble at dimensionless temperature $T = 1/2\pi$ with

⁶Ted refers to [3].

⁷Actually, the "first law" should, I think, be the statement of energy conservation in thermodynamics. In this relation dE refers to the heat δQ added to the system, and I don't know a name for the relation $\delta Q = TdS$! Nevertheless, people seem always to call this the first law in the context of black hole thermodynamics, so let's keep doing so.

respect to the dimensionless boost Hamiltonian (i.e. the generator of Lorentz boosts). This theorem holds for any interacting field theory on a Minkowski background. The proofs of this fact are very deep, relying only on stability of the vacuum and local Lorentz symmetry, so I would assert that there is good reason to think that it extends to quantum gravity. The system of quantum field fluctuations at short distances outside a local Rindler horizon is well described by a canonical ensemble, and it is the collection of these systems outside a black hole horizon that I propose to identify with the “surface system” of a black hole. The entropy of this ensemble is the entanglement entropy of the vacuum, and is divergent in ordinary local quantum field theory due to the absence of a UV cutoff. I suppose that it is somehow rendered finite in quantum gravity.

Now suppose some energy crosses the black hole horizon. It is the thermodynamic description used by the outside observers that interests us. These observers never see the energy cross the horizon, they just see it approach ever closer. That is, they see it sink into the surface system. Since the surface system is initially described by a canonical ensemble, and since the energy crossing the horizon is assumed to make only a very small perturbation of the vacuum at short distances, the absorption of this energy into the surface system is associated with an increase of entropy $\delta S = \delta E/T$. Here E is the mean value of the boost Hamiltonian, and T is the dimensionless temperature $1/2\pi$. As pointed out in your paper⁸, the variation of the entropy is finite even though the entropy itself is UV divergent. One can impose a UV cutoff, compute the variation, and take the cutoff away, which yields a variation independent of the cutoff. Even with a cutoff in place, one would obtain the same result, provided the cutoff is at a large enough energy.

So far I’ve just used the local Rindler horizon approximation to a patch of a black hole horizon. Now I put together such patches to cover the black hole horizon, rescaling the boost Killing vector in each patch to agree with the black hole horizon generating Killing vector. This also rescales the boost Hamiltonian and temperature by the surface gravity of the black hole so that they agree with the Killing energy and Hawking temperature. (In the case of a rotating black hole it is actually the Killing energy minus the Killing angular momentum times the angular velocity of the horizon.) This yields the relation $\delta S = \delta E_{BH}/T_H$, where E_{BH} is the black hole energy and T_H is the Hawking temperature. The argument up to here is, I think, essentially equivalent to

⁸Ted again refers to [3].

the one spelled out in detail by Zurek and Thorne using the hypersurfaces on your left hand diagram.⁹ This is not yet a derivation of the first law for black holes, since in that law what appears is the variation of Bekenstein-Hawking entropy $S_{BH} = A/4\ell_{\text{Planck}}^2$, not the variation of entanglement entropy.

But we are almost there. As I said before, the entanglement entropy is proportional to the surface area, with a proportionality constant η that seems quite plausibly universal, since all local Rindler horizons are equivalent. Thus it only remains to explain why $\eta = 1/4\ell_{\text{Planck}}^2$. One might just refer to the classical geometric derivation of the first law in general relativity, and “read off” this relation by comparison with the statistical first law, as did Zurek and Thorne. But I think this would not be fulfilling your call for a truly statistical derivation of the first law for black holes. To establish this link between η and the Planck length, I will invoke my derivation of the Einstein equation as a thermodynamic equation of state¹⁰. In that paper, I showed that the validity of the first law for an entropy of the form $S = \eta A$ at all local Rindler horizons implies that the Einstein equation *must* hold, with Newton’s constant, or rather $\hbar G = \ell_{\text{Planck}}^2$, equal to $1/4\eta$.

I believe that this line of reasoning not only provides a microscopic derivation of the first law of black hole thermodynamics, but it makes abundantly evident the fact that the interior of the black hole is irrelevant to the story.

Don: Thanks, Ted. I think we are now moving to the heart of the matter.... For me, the most important thing you just said was

“It is the thermodynamic description used by the outside observers that interests us. These observers never see the energy cross the horizon, they just see it approach ever closer. That is, they see it sink into the surface system.”

So then, it seems that we have now changed the framework of the discussion to one associated with the *left* diagram that I drew. (*Don points again at Figure 1, and taps the left-hand diagram demonstratively.*) In this setting I could indeed imagine a derivation of the first law. But now what does it mean to say that your surface system does *not* describe the entire black hole, but that in addition there is a separate interior system?

⁹Ted refers to [4].

¹⁰Ted refers to [5].

In particular, doesn't the data near the horizon determine the interior of the black hole? And doesn't this mean that your surface system must have at least as many states as the black hole interior?

Ted: Hmmm...mmm...hold on, no. The statement that outside observers never see the energy cross the horizon does not entail any choice of surfaces. And I really don't want to use the surfaces that never cross the horizon, since on them there is no entanglement entropy.

Let's call the surfaces on the left "static surfaces", and those on the right "horizon crossing surfaces". It is true that a packet of infalling energy does disappear from the exterior portion of the horizon crossing surfaces after some time. However, this presents no problem for my derivation of the first law, so what are you concerned about?

Don: Perhaps my asking about hypersurfaces was not helpful, but let me try to phrase the question another way. Consider a process in which an energy dE is added to the black hole. It is reasonable that this energy first enters your surface system, but the question is then about what happens next: Does the energy dE remain within the surface system and thermalize there? Or does part (or all) of the energy enter the interior?

Now, back when you referred to a description used by outside observers, I thought that you wished to say that the energy dE remained in the surface system. In this context, I find it plausible that the energy might thermalize and lead to $dE = TdS_{surface}$. Mind you, this thermalization does not appear to occur classically, but I am happy to interpret the results of, for example, my paper with Minic and Ross that you mentioned earlier as evidence that such a thermalization does occur quantum mechanically.

However, if I use a description where all of the energy added to the black hole stays within the surface system, then I don't see a sense in which I can talk about the black hole interior as a separate system carrying additional degrees of freedom. Can you fill me in?

Ted: What seems to bother you is that you think I claim that on the one hand the energy dE thermalizes in the surface system, but on the other hand it crosses the horizon. This sounds impossible, since to say that energy thermalizes is to say it is distributed over a large number of a specified set of degrees of freedom, which locates the energy in those degrees of freedom, and you think I am saying that the energy is both in the surface system

degrees of freedom and in the interior degrees of freedom at the same time (i.e. on the same horizon crossing slice), which is a contradiction since the energy cannot be in two places at once. Is that an accurate description of your objection?

Don: Yep.

Ted: Well if you look back you'll see that I never said the energy thermalizes in the surface system! What I said, just after what you previously quoted, was

“Since the surface system is initially described by a canonical ensemble, and since the energy crossing the horizon is assumed to make only a very small perturbation of the vacuum at short distances, the absorption of this energy into the surface system is associated with an increase of entropy $\delta S = \delta E/T$. Here E is the mean value of the boost Hamiltonian, and T is the dimensionless temperature $1/2\pi$.”

The subtlety here is that the energy may be macroscopic, e.g. a pebble, while the entropy change δS is carried by the near Planck scale degrees of freedom. I am not asserting that the pebble mass is spread over these degrees of freedom. So “thermalization” of the pebble in the sense you were concerned about does not, and need not, occur in order for me to invoke the formula $\delta S = \delta E/T$.

Now you might object that, in the description I wish to use, the pebble energy does not remain in the surface system, but rather crosses the horizon into the black hole. And this suggests that the surface system has then *lost* the pebble energy. So, you might well ask, why does the surface system entropy not go back down to where it started before the pebble arrived?

I claim that the answer lies in gravity. In a non-gravitational, flat space-time setting, the pebble would pass through the Rindler horizon surface system and out the other side, and the entropy would go back down. But you *cannot* turn off gravity. The process of the pebble falling into the black hole is associated with an increase in the mass of the black hole, which therefore acquires a slightly larger horizon area.

In a bit more detail, the relation between the pebble energy and the surface system equilibrium is as follows. The pebble is much wider in the radial direction than the surface system, so only a very small part of the

pebble energy resides in the surface system at any given time. The passage of pebble energy makes a small, adiabatic perturbation on the surface system, increasing the entropy. When that energy exits the surface system and crosses the horizon, it leaves behind an entropy imprint via the increase of horizon area. The surface system is then in a new canonical ensemble, since it is a slice of the near-horizon vacuum on the background geometry of a slightly larger black hole. Hence the increase of surface system entropy is for keeps (unless we let the black hole evaporate), even though the pebble energy falls across the horizon and does not remain in the surface system.

Don: Very good, Ted. I now see your picture much more clearly. But do you think that your arguments above really constitute a *derivation* of the first law from a complete statistical mechanical picture? Or, at this stage are you instead *using* the classical first law to deduce that “somehow” gravity leads to the scenario for “thermalization” described above in which the surface system gets the entropy “for keeps”?

Ted: I think the argument really constitutes a derivation of the first law from a statistical mechanical picture. The fact that classical gravity enters in a critical way to “preserve” the entropy change in the surface system is just a peculiar feature of this physical system, but does not make the derivation any less statistical mechanical.

Don: Well didn’t you appeal earlier to your derivation of the Einstein equation *from* the classical first law in order to relate the entropy change of the surface system to the change of the Bekenstein-Hawking entropy of the black hole?

Ted: Remember, the statistical argument gave me a first law involving the entropy change $\eta \delta A$. I then argued that this is consistent for all local Rindler horizons only if there is gravity and it acts according to the Einstein equation with Newton’s constant equal to $1/4\hbar\eta$. So I don’t think I’ve assumed that the first law holds. I’ve argued statistically that it holds with entropy ηA , and inferred from that the existence of gravity with the above Newton constant. Thus I *deduce* that this is in fact the Bekenstein-Hawking entropy.

Don: OK, let me see if I’ve got the argument now. You first *assume* that your model is consistent, and in particular that there must be some

mechanism to (permanently) increase the entropy (and thus the area) of your local Rindler horizon. Given this assumption, it then follows that this effect must be the same as one would derive from the gravity and the Raychaudhuri equation. In fact, you actually get all of the Einstein equations from this argument!

Ted: I hadn't realized so, but yes, I have assumed that the entropy increase is permanent (neglecting Hawking radiation). The area increase on the other hand is not an assumption but is inferred.

Perhaps the argument can be improved, however. It seems the permanence of the entropy increase of the local Rindler horizon is *required* if the second law of thermodynamics is to hold for the outside observers! Such a version of the second law would refer to the total entropy of the exterior, including the surface system. Let's call this the "exterior second law", or "ESL" for short.

Don: Aha! So, you wish to kill two birds with one stone! Well, one concern I have here is that, at least in standard statistical mechanics, the first and second laws have a very different microscopic character. The second law of course holds only in a statistical sense, whereas the first law is a precise statement about ensembles. That is, if a warm lump of coal happens to undergo a fluctuation to a lower entropy state (in a microscopic violation of the second law), then this is certainly accompanied by the emission of a small bit of energy of magnitude roughly TdS (and thus satisfying the first law)..... But this is a fine enough point that I suppose you can claim that both features will be enforced by the same microscopic mechanism.

On the other hand, I find it hard to see how *any* microscopic mechanism would enforce the second law in your setting... I mean, we don't usually say that the region *outside* of a black hole satisfies the second law at all. Rather, we say that we have a *generalized* second law, where entropy falling across the horizon leads to an increase in black hole entropy. I realize that in your scenario you want the generalized second law to become the regular second law for your surface system together with the exterior.... But what I want to emphasize is that you need the surface plus exterior system to satisfy the second law in the form $\delta S \geq 0$ usually reserved for *closed* systems. And you need this despite agreeing that energy can pass into the interior, so that this system looks to me to be an *open* system, which would naturally satisfy a second law of the form $\delta S \geq Q/T$, which allows S to decrease when Q is

negative; i.e., when heat leaves the system. I emphasize that this weaker form of the second law would not be sufficient for your purposes.

Ted: It’s interesting how we have arrived at the very issue that led to the birth of black hole thermodynamics. I think it was precisely the concern that the second law would be violated for outside observers that led Bekenstein to propose that black holes have entropy! While the exterior is an open system in the sense that things can leave it, it is self-contained dynamically (since the horizon is a causal barrier), which is not true for a generic open system. If the ESL did *not* hold, I’d expect that one could make a perpetual motion machine exploiting that.

It has often been remarked that a failure of the *generalized* second law—for black holes or even for de Sitter spacetime—would lead to the possibility of constructing a perpetual mobile. Such remarks assume that the Bekenstein-Hawking entropy S_{BH} is the balance of the “real” entropy in the system, so that standard thermodynamic reasoning can be applied. I am saying the same thing, but in place of S_{BH} I have the surface contribution to the entropy visible to the outside observers. Since the surface contribution *is* the balance of real entropy in the system accessible to the outside observers, the perpetual mobile argument should be valid for the ESL.

Don: I’m quite skeptical ... So, I’d really like to see you try to write down the details.... In any case, I think you are implicitly making another assumption, namely, that the horizon is a strict causal barrier even at the quantum level, so that no entropy from inside can ever influence the exterior.

Ted: Yes. it is an underlying assumption of my picture that no information from inside can influence the exterior. In this setting, I think the perpetual mobile argument strongly supports the ESL, although it is not the *microscopic* account you are asking for. Rafael Sorkin has thought a lot about precisely this question, and has offered an outline of a microscopic proof in both the semi-classical and full quantum gravity settings.¹¹ I think there are gaps in the outline, but maybe he’s on the right track.

Don: Maybe, but maybe not ...

Well, Ted, even after all of this discussion I’m afraid I still find it very hard to see how any microscopic mechanism could allow your surface system

¹¹The ideas are explained in Refs. [1, 6].

to “keep” the entropy associated with the passage of a bit of energy dE after this energy has passed on through the horizon. I guess we’ll have to leave the issue here for now, but I would have to see this sorted out in detail before I could really accept your point of view.

Ted: I’m not sure if *I* accept my point of view. Your questioning has forced me to sharpen my views about the nature of black hole entropy into a more definite picture. For the moment, it makes complete sense to me, and in particular I think it gives a cogent answer to why the surface system keeps the entropy. But our discussion has certainly given me much food for thought.

Don: Well, then, I am sure we will discuss this again! But, speaking of food, it looks like it’s now time for dinner. Shall we adjourn to the dining room? Maybe we can get Carlo to join us...

Scene II: Horizon fluctuations over dinner

Don and Ted join Carlo at the dinner table. The aromas of a Provencal meal in preparation waft in from the kitchen.

Don: Hello, Carlo! I’ve been grilling Ted over his views on black holes and the idea that the Bekenstein-Hawking entropy resides in a set of surface states. In particular, I’ve been asking how he would derive the first law in this picture. You are also known to expound this view, so I’d love to get your answer as well!

Carlo: Indeed, I was hoping to catch you! I was listening to your conversation earlier on the terrace, but I did not want to break your line of discussion. Now it seems that I have my chance!

My general understanding of the situation is very much in line with Ted's. I agree with Ted that we can understand the BH entropy in terms of degrees of freedom sitting on the surface. The pebble falls across the horizon and leaves an ensemble with higher entropy. I also agree with Ted that we can conceptually identify three subsystems here: the exterior, the surface and the interior, and that the key of the story is in the peculiar property of the interaction between the surface and the interior: the evolution of the interior depends on the surface, but not viceversa: the evolution of the state of the surface (and the exterior) does not depend on the state of the interior.

Ted: So far, quite good...

Carlo: But there are also differences in the way you and I look at the problem, Ted ...

Ted: No doubt!

Carlo: Your picture is in terms of fields on a spacetime background. This spacetime background is taken to be non-fluctuating neither quantum mechanically nor thermally. Such a picture is necessarily approximate. I want to consider a picture which is physically more complete, in which we take into account that the spacetime geometry fluctuates. I think that this can provide a key ingredient to better understand the entropy of the hole.

The spacetime geometry fluctuates for two reasons. One is quantum mechanics, the other is just because at finite temperature *all* dynamical quantities undergo statistical fluctuations. The two kind of fluctuations might perhaps even be related. But even disregarding quantum fluctuations, spacetime geometry must fluctuate thermally in a hot environment. Therefore it is not realistic to assume a given fixed background geometry, such as, say, a Schwarzschild background.

Ted: I don't think we need to address these questions for the understanding of black hole thermodynamics, since the setting is that of a black hole very large compared to the Planck length. We need only that the leading order term in the entanglement entropy scales with the area, with a universal coefficient. All other details are strictly beyond the purview of our current understanding of quantum gravity, and beyond the theoretical resolution needed to understand the origin of black hole thermodynamics.

Carlo: Maybe. But geometry fluctuations *are* there. I think that, if we want to do statistical mechanics in a context in which gravity plays a central role, it may not hurt to actually consider the statistical fluctuations of the gravitational field, namely of the geometry. Notice that this can be done irrespective of whether we think of the metric as a fundamental variable or as a collective variable. About quantum gravity, I respect your view that the quantum gravitational picture of the situation is beyond the purview of our current understanding of quantum gravity, but I do not share it! However, this is not the subject of our discussion!

Ted: Okay, let's consider the fluctuations of the geometry. Then?

Carlo: Then, the geometry of the horizon fluctuates. The horizon is in a thermal mixture of different positions, or different shapes. The horizon is never a perfect sphere, but rather a closed surface that fluctuates over an ensemble of shapes. I think that the degrees of freedom responsible for the entropy must be precisely the ones characterizing the geometry of the horizon itself, namely the *shape* of the horizon.

Ted: The horizon is by definition the causal boundary of the past of future null infinity. The fluctuations in that boundary today are determined by the entire future. So how can one meaningfully talk about the shape of the horizon at one time as a dynamical variable that can carry entropy?

Carlo: I think that a clean description of the system is atemporal: the statistical mechanics of a relativistic gravitational system is defined classically by an *ensemble of solutions of the field equations*. If we then choose a foliation in each, and (for the sake of the analysis) identify equal time surfaces, then we get an ensemble of “horizon shapes” for each coordinate time t .

Don: This sounds quite different from Ted's picture. You say that the degrees of freedom counted by the BH entropy are the ones characterizing the shape of the horizon. Ted says it is the entanglement entropy of quantum fields around the horizon.

Carlo: Yes, this is indeed a key difference I wanted to point out. The picture that I find more compelling¹² is in a background independent context,

¹²Carlo refers to [7].

Ted's is in the non-fluctuating background approximation. The two pictures are not necessarily incompatible. Suppose you accept this idea that the degrees of freedom counted by the BH entropy are the ones characterizing the shape of the horizon. But you decide to describe the gravitational field in terms of a background plus a fluctuation. Then the quantum fluctuations of the shape of the horizon get reinterpreted simply as the fluctuations of the gravitational field in the vicinity of the horizon, which is what Ted wants to identify as the source of the entropy.

Ted: At the level of effective field theory, certainly the gravitational field fluctuations are a part of the overall set of interacting degrees of freedom. Also any matter field fluctuation drives a metric fluctuation, so they are inextricably linked, especially at the Planck scale where most of the entropy presumably resides.

Carlo: Yes!

Ted: So, I agree that the horizon fluctuations should be included as *a* source of black hole entropy, but I don't think they are *the only* source. I don't see how it could be correct to attribute *all* of the entropy to just the fluctuations of the horizon geometry.

Carlo: Mmmm... my reason to limit my consideration to these is double: first, a vague intuition that gravity ends up dominating at very small scale; second, the quantum gravity calculations that appear to be sufficient to give the correct entropy. But I am happy to leave the issue of whether the matter contributions are of the same order open...

Don: Good. Let's put aside for now the question of the identification of the surface degrees of freedom—shape of the surface (and thus purely gravitational) for Carlo, or entanglement entropy (of all fields) for Ted—and of their relations, and let us get back to my question about the first law.

Carlo: Perfect. It seems to me that the core of your question, Don, is the following: if the surface degrees of freedom thermalize, is equilibrium reached by sharing the energy over the surface degrees of freedom alone, or rather over the degrees of freedom of surface and interior? Is this your question?

Don: Yes, indeed.

Carlo: Your point, if I understand correctly, is that if the surface thermalizes together with the interior, then the relevant number of degrees of freedom is the ones of the coupled surface+interior system. But if the surface thermalizes just by itself, then no information is lost inside. Right?

Don: Yes, and in particular, in the second case I don't really understand what it means to say that there is a separate interior system with additional degrees of freedom.

Carlo: And you object that, contrary to what Ted and I think, the “first” law of thermodynamics $dS = dE/T$ cannot be derived, if S is determined by the number of surface states alone. Is this a fair characterization of the disagreement?

Don: Yes, Carlo, that's it exactly.

Carlo: Well, I think that, to use your expression, we can have our cake, and eat it too. So, let me begin by sketching a picture of a statistical interpretation of BH entropy. I may refine it and fill missing steps in as we go along.

I want to analyze the problem classically, namely disregarding quantum theory. I will only bring some quantum effects into the picture later on, when they are relevant. This, let me say upfront, is the main weakness of the picture. I wish I had a fully quantum picture, but I do not, for reasons that will be clear in a minute.

So, in classical GR (plus any matter you'd like), let me call Γ a global ensemble of states of the system, and Γ_i , Γ_s and Γ_e the ensemble of the possible states of the interior, surface and exterior. As I said before, Γ is an ensemble of solutions of the classical field equation. It is an a-temporal statistical state.

If we fix a one-parameter family of horizon-crossing Cauchy surfaces (like the ones in Don's picture on the right) labeled by a parameter t , then each (atemporal) state in Γ defines the three evolving states $s_i(t)$, $s_s(t)$ and $s_e(t)$. Now, the future evolution of $s_s(t)$ and $s_e(t)$ is independent from $s_i(t)$, but not the past evolution. Hence there is information loss (we have the cake).

Now, the “equilibrium states” are the ones for which certain macroscopic parameters do not change with t . Suppose at some t we have a certain (energy) macroscopic parameter E_e in the exterior, and a certain (energy-like)

macroscopic parameter E_s on the surface. I consider microcanonical statistical ensembles defined by all the microstates in which these macroscopic parameters have a given value. I assume an ergodic hypothesis that all processes are allowed, in the sense that the t evolution drives the evolving states all over this microcanonical ensemble. A given value of E_s defines the surface statistical state.

Next, let the number of surface plus exterior states characterized by E_s and E_e be N . By ergodicity, $N = N_e(E_e) \times N_s(E_s)$, where $N_e(E_e)$ is the number of states of the exterior with Energy E_e and similarly for $N_s(E_s)$. Consider a different macroscopic equilibrium state obtained by an adiabatic process that decreases E_e by dE and increases E_s by a corresponding amount. The corresponding number of microstates will be $N(dE) = N_e(E_e - dE) \times N_s(E_s + dE)$. If our original state is the most probable, N must be the maximum of $N(dE)$, obtained for $dE = 0$. Hence $\frac{d}{dE} \log N(E) = 0$. Namely $\frac{dS_e(E_e)}{dE_e} = \frac{dS_s(E_s)}{dE_s} \equiv \frac{1}{T}$. That is, an equilibrium situation is characterized by a macroscopic parameter T which must be the same for the exterior and the surface and such that $dE = TdS$. Thus, it seems to me that this relation can be derived without conflict with the fact that information is lost to the interior. We can have the cake, and eat it...

Ted: Well, Carlo, you have just reiterated the standard stat mech treatment of equilibration between two systems, in this case the surface and the exterior.

Carlo: Essentially, yes. But carefully choosing a version which is sufficiently general to be used in a generally covariant context and adapted to our case...

Ted: Okay, but I see several problems with this line of reasoning, since the assumptions of the standard argument don't seem to hold here. First, you said the exterior loses energy dE and the surface gains dE . This assumes we are in a setting where two systems are exchanging energy and total energy is conserved. But you started out saying that the surface parameter E_s is "energy-like". Since there is no conservation law for $E_e + E_s$, your calculation of the condition for maximizing the number of states seems to me unjustified.

Second, the first law for black holes relates the *energy* change of the black hole to its Hawking temperature. How are you characterizing the energy of the black hole? And how are you relating the T in your equation to the Hawking temperature?

And third, your argument requires that if I throw energy dE into a black hole, the states of the exterior with energy $E - dE$ are equiprobable. But this assumes the exterior is in equilibrium with this energy. Why should that be so? A pebble falling into a black hole does not look like an equilibrium state. In my argument, the equilibrium pertains to the vacuum fluctuations at short distances. You are eschewing the role of these fluctuations in your account, but then how do you account for the equilibrium you are invoking?

Carlo: Very good. This takes us to the center of the discussion and connects to what the two of you were discussing earlier on the terrace. Seems to me that the delicate points in your picture, vulnerable to Don’s criticisms were precisely connected to this issue. Doesn’t the energy dE that enters the horizon immediately cross over to the interior? And then how can it contribute to the increase $dS = dE/T$ of the entropy of the horizon’s degrees of freedom? This was one of Don’s main lines of attack. Your answer made reference, I believe correctly, to the specific features of the gravitational dynamics.

Ted: Yes, but as I said before after the relentless questioning by Don, I don’t need to assume that dynamics. I need assume only the second law of thermodynamics, and I deduce the gravitational dynamics.

Carlo: Good. The picture I am discussing is different because I take a different starting point. Let me first sharpen what I just said.

You say, very truly, that there is no energy conservation that allows us to follow the usual statistical mechanical line of thinking. In fact, a strict attachment to the nonrelativistic notion of energy is not useful in the general relativistic context, where energy is a more delicate notion. Now, what is the role of energy conservation in the microcanonical setting? It is to reduce the region of phase space where the system is free to wander (ergodicity then tells us the systems are all over the allowed region). Now, in general relativity there is no analogous energy conservation. Thus, we have to look for a dynamical input from the Einstein equation that can play the same role that energy conservation plays in the nonrelativistic context. We have precisely what we need: the theorem stating that *“the area of the horizon does not decrease”*. This theorem captures the relevant information from the general relativistic dynamics needed for understanding the statistical mechanics of the horizon.

Now, if the area cannot decrease, and if we are, as we assumed, in a statistical equilibrium configuration (where everything that microscopically changes will later change back), then the only possibility is that the ensemble fills a region of constant area. In other words, a microcanonical ensemble determined by the value of the macroscopic area parameter is singled out by the Einstein dynamics. This is what replaces the role of energy conservation in the usual nonrelativistic microcanonical ensemble.

Notice that I have not proven that the surface degrees of freedom do in fact thermalize. But this is *always* horrendously difficult in statistical mechanics! We can just follow the usual argument that under some ergodicity hypothesis tells us what are the equilibrium states, if they exist. In statistical mechanics, the relation $dS = dE/T$ is derived as a relation between nearby equilibrium states, not as a description of a dynamical process of approach to equilibrium...

What I am saying is that the area is the only quantity that (classically) cannot decrease, and therefore the ensemble is formed by all the microstates with a certain area. The Einstein equation tells us also that if we throw a certain amount of energy dE across the horizon, then the area will increase by a certain amount. Hence a change in area is connected to a change in external energy. If we throw an energy dE inside the hole, then its horizon will fluctuate over a different ensemble, characterized by a larger area. There is no need to think that there is an actual “Energy” that remains on the surface degrees of freedom without continuing inside the hole. The Einstein dynamics allows us to apply the usual statistical mechanics logic, with just what is needed: a condition on the set of microstates in which the system can be. I think that this is an answer to Don’s question about the fact that energy enters the hole and therefore cannot thermalize on the surface degrees of freedom...

Ted: If I understand correctly, you are saying that you need not invoke energy conservation, since its only role is to fix the region of phase space in which the system can wander, and you propose that, because of its classical non-decreasing property, the horizon area is suitable for playing that role in the surface system...

Carlo: Yes.

Ted: But Carlo, I don’t see how the classical area theorem can consistently motivate a classical fixed area microcanonical ensemble. The reason

is that the fluctuations which you certainly expect to have in your ensemble are certainly not static, yet it is only in the static limit that there is no area increase: the slightest amount of gravitational radiation, for example, produces a Weyl tensor component that generates shear and thus expansion of the horizon generators.

I'm not saying that I don't believe the horizon area is fluctuating with some mean value determined by the macroscopic parameters—I do. But I don't think this is related to the classical area theorem, nor do I think you have justified your microscopic picture of the horizon as a “system” in a certain dynamically selected ensemble.

Don: I have a related objection, Carlo. Your statistical argument considered the surface system together with the exterior, and in particular it considered the most entropic state of the joint system formed by taking the surface system and exterior together. What I do not see is how you justify that the joint system should be in the most entropic state, if you stay strictly in the classical domain.

In standard statistical mechanics one would justify maximizing the entropy of this joint system by considering all possible fluctuations and arguing that, most of the time, the system remains at or near the state maximizing the entropy. However, your argument uses the area theorem, which requires that the area never decrease! Thus, if there is ever a fluctuation to a state in which the area is *greater* than occurs in the most entropic state, doesn't your argument forbid the system from ever returning to the most entropic state? So then, why should the most entropic state (of the surface and exterior together) characterize this system?

Carlo: Maybe I have over-emphasized the role of the classical second law in determining the ensemble. The question is whether in the limit in which we disregard quantum effects, the area is compatible with the Einstein dynamics, as a parameter characterizing an equilibrium state. The answer is yes in the sense that a change of area is irreversible in classical GR. Hence an ensemble can be in equilibrium only if it is at fixed area. Maybe a classical equilibrium ensemble is necessarily trivial, namely formed by a single state, as your intuition suggests. I am not sure, but even if you are right, this is not the point here. The ensemble I am interested in, is not in the classical theory.

I know I am zigzagging between the classical theory and bits of informa-

tion about quantum effects. Maybe this zigzagging is nonsense. But this is the best I understand of black hole statistical physics. I think and hope that a fully quantum treatment could fully justify the legitimacy of this semiclassical thinking, but I cannot prove this.

Ted: OK, well let's put this question aside, and continue the discussion of your derivation of the first law that you sketched previously. It seems crucial there that (i) an *energy* dE is being transferred from the exterior to the surface system, and (ii) the *energy* of the exterior decreases by the same amount that the *energy* of the surface system increases. It is only this way that the equality $\frac{dS_e(E_e)}{dE_e} = \frac{dS_s(E_s)}{dE_s} \equiv \frac{1}{T}$ follows, which allowed you to identify the macroscopic parameter T and its role in the first law for the surface system. But E_s is not energy, and $E_e + E_s$ is not conserved. So, while I appreciate your later *description* of the first law in terms of the change of ensemble over which the horizon fluctuates, I don't see how you are offering a *microscopic derivation* of this law as Don requested.

Carlo: What I need to do, as I argued before, is to replace E_s , which is what I called the “energy-like” macroscopic parameter of the surface, with the area A . Then I call $S_s(A)$ the logarithm of the number of states with area A . The input I need from the Einstein equation is that I can increase A by an amount dA by sending an amount dE of external energy over the horizon. I know this happens for one particular microstate: Schwarzschild. Therefore I know I can go from one equilibrium state to the next by using this amount of external energy. The relation between dA and dE in the case of Schwarzschild is obtained from the relation between the energy E , or mass $M = E$, of the quasi-isolated hole and its area $A = 16\pi G^2 E^2$. Hence $dA = 32\pi G^2 E dE$. Thus, we can say that an equilibrium statistical state with external energy $E - dE$ corresponds to the microcanonical surface ensemble characterized by the macroscopical parameter $A + dA = A + 32\pi M G^2 dE$. Using the line of argument above (which, as you said, is an adaptation of a standard microcanonical argument), we have then that in order for the system to be in a maximally probable state, the energy transfer should not increase the total number of states, hence there must exist a quantity T such that $\frac{dS_e(E_e)}{dE_e} = 32\pi M G^2 \frac{dS_s(A)}{dA} \equiv \frac{1}{T}$. Hence we have $dS = dA/(32\pi M G^2 T)$. Isn't this the first law?

Of course, in order to have the relation between this T and A , we have to know something else. Either we use the indirect result by Hawking that

$T = \hbar/8\pi GM$ or, if you have some confidence (as I do) in a quantum theory of gravity where you can actually count the number of surface states with given area, you can directly obtain this relation from the knowledge of the function $S(A)$.

There are various weaknesses...but to me this sounds like a credible story for thinking about the black hole entropy in terms of surface states in a fully general covariant setting, and without being vulnerable to Don's tough criticism...

Don: Well, Carlo, I still have several other questions... For the first one, let's return to your "replacement" of E_s by the area. Classically, it is true that if I limit myself to moving from one Schwarzschild black hole to another, then indeed dE and dA are related as you state above. But more generally this relation is modified. For example, for Kerr black holes there is also a term ΩdJ . And what if we consider a static black hole which is distorted by some external matter, such as a steel cage built around the black hole? Then surely the relation depends on many other parameters that describe this cage...

Carlo: The essential point of my discussion is the distinction between the macrostate and the microstates. So, you are actually posing two different questions here. The first question refers to the macrostate. Namely, you are asking what happens if I am dealing for instance with a hole that has a *macroscopic* angular momentum. Or is *macroscopically* distorted, as a hole in a cage.

Don: Actually, I would like to ask this question at both levels, but let us discuss the macroscopic level first.

Carlo: Perfect. Well, as is always the case in thermodynamics, if you describe a system characterized by more macroscopic parameters, such a rotating or distorted balloon, you'll have a different ensemble. So, in a rotating or distorted black hole, the ensemble will be described by more parameters than just the area. I haven't worked out these cases. There is recent work in this direction.¹³ Here, for simplicity, will you allow me to consider just spherical equilibrium states for the rest of our discussion? It is like doing the statistical mechanics of the gas in a spherical balloon.

¹³Carlo refers to [8].

Don: Fine, but now let us move on to the microscopic version of this question ... Do you require all of your microstates to be spherical? Birkoff's theorem helps you a lot in this case...

Carlo: (*Much agitating hands in Italian way*) Not at all! Not at all! In a spherical balloon the motions of the molecules are *not* spherical! If you try to work out the statistical mechanics of a spherical balloon of gas by restricting to spherical motions of the molecules you get it all wrong, of course. The fact that the hole is *macroscopically* spherical does not imply that its fluctuations are spherical!

Don: Very good! So this takes us back to my question: for the large majority of your microstates dE and dA are not related as you state above.

Carlo: You are right! Here is a subtle point. Suppose I have a system described by an ensemble of area A and I want to move it to an ensemble of area $A + dA$. How much heat do I need? Well, I do know that there is at least one path from one microstate in the first ensemble (Schwarzschild) to one microstate in the second ensemble (Schwarzschild with larger area) that can be taken with cost $dE = dA/32\pi MG^2$. Hence I know that this amount of heat is sufficient to go from one ensemble to the other.

Don: OK, I am still confused. We agree that the change dA in the horizon area depends on how you remove an energy dE from the exterior ... So, it looks to me like adding a given amount of "heat" to one of your ensembles of fixed area does *not* lead to another ensemble of fixed area ... Isn't this a problem?

Carlo: I am not saying that each microstate in the first ensemble becomes a microstate of the second ensemble by means of the same external dE . I am saying that there is one microstate with this property. Therefore I expect that there exists an adiabatic process going from one ensemble to the other with a cost dE in external heat.

Don: Hmmm ... I must say that my own expectations differ markedly ... but let's move on another of my questions.

Why, in your picture, does one maximize the joint probability only for the surface system and exterior, without regard for the interior. Don't we in general need to include all systems which exchange, in your language,

“energy-like quantities”? At least quantum mechanically via Hawking radiation, energy certainly flows back and forth between the surface system and the interior, so doesn’t this indicate that we should include the interior of the black hole as well?

Carlo: The condition $dS(E_1)/dE_1 = dS(E_2)/dE_2 = 1/T$ between two systems at equilibrium holds just because of the interactions between these two systems.

Don: Well, there is still the important question of whether $dE_1 = dE_2$... But I see that you expect the “quantum first law of black hole mechanics” to take care of this issue... Of course, I would like to see the details ...

Carlo: Yes, I am using a classical result, while I am already outside the classical picture... This is again the zigzagging between classical and quantum I just referred to... The entire picture makes sense only under the unproven hypothesis that this zigzagging is legitimate...

Don: OK, I think I now have a much clearer understanding of your picture. However, I see that you have fallen back on logic much like Ted’s, where you assume at least the classical first law of black hole *mechanics*, which is what I had implicitly hoped that you would derive.

Ted: I object, your honor! The logic is most certainly *not* much like mine! I *derived* the first law of black hole mechanics, and the Einstein equation along with it!

Don: Calm down now, Ted. We’ve already agreed that you have postulated a mechanism (whose existence you believe is associated with the second law of thermodynamics, but whose microscopic description you do not understand) to allow your surface system to “keep” the entropy it picked up previously from a bit of energy that was just passing through... But I’ll retract my reference to your logic so we can focus on the issue at hand, which for now is Carlo’s use of the first law of black hole mechanics.

Carlo: The story I am suggesting is precisely the microscopic mechanism to allow the surface to “keep” the entropy it picks up when a bit of energy passes through: when the bit of energy passes through, the surface keeps

fluctuating statistically, but over an ensemble of surfaces of larger area, because this is the only possibility consistent with the Einstein equation and the assumptions about macroscopic equilibrium.

But there is a major difference between what Ted does and what I am doing. Ted has this marvelous acrobatics that has allowed him to actually *derive* the Einstein equation thermodynamically. When he did this a few years ago, I remained astonished, and I haven't yet digested the result.

I myself am far more conservative: I use the very standard logic of statistical mechanics, where you assume mechanical laws, and use statistical arguments to infer the thermodynamical ones. Since gravity is involved, and its degrees of freedom are part of the story, the relevant mechanical law is given by the Einstein equation.

Therefore of course I use the laws of hole mechanics! These laws are just a consequence of the dynamics of the theory, of the Einstein equation. In fact, they are *the* property from which the thermodynamical behavior follows. They play the same role as the microscopic energy conservation in the usual nonrelativistic thermodynamic context. I did not think that you wanted us to derive *this* law!

Don: Actually, I did... You see, I was looking for something more here...

As has often been noted, the laws of black hole mechanics strongly resemble the laws of thermodynamics. And, one of the greatest advances of physics was to understand that the laws of thermodynamics follow (more or less) from elementary statistical mechanics. Thus, the laws of black hole mechanics are typically taken to suggest underlying statistical properties of black holes... At least in my own view, this naturally suggests that one should be able to use stat mech to derive the first law of black hole mechanics *without* first assuming the Einstein equation! You may feel that this is too much to ask, either in principle, or at least at our current state of knowledge, but it is a goal to which I think we should aspire!

Carlo: I think that you are mixing two different issues here. One issue is whether a theorem that follows from the Einstein equation can be derived from some statistical argument. I have nothing to say about that. Ted might see further than me here, but at the present stage of knowledge, I prefer to take the gravitational field as a field, whose mechanics is described, at least within some approximation, by the Einstein equation. The other issue is whether or not entropy counts surface states and how to derive $dS = dE/T$

from the surface state picture. This second issue is what I have discussed. The two issues are quite distinct.

Don: Perhaps this suggests the crucial difference between our points of view. Of course the *classical* first law of black hole mechanics follows directly from the classical Einstein equation. The same is true of the Hawking area theorem; i.e., of the classical second law of black hole *mechanics*. But we know that the latter fails quantum mechanically, as the area will decrease during Hawking radiation. However, because black hole entropy dominates over any other form of entropy as $\hbar \rightarrow 0$ we can naturally *explain* the Hawking area theorem as the classical limit of a second law of thermodynamics. Such an explanation would work even if the quantum dynamical laws are very different from the classical Einstein equations (perhaps in analogy with the situation for hydrodynamics). To me, it seems natural to seek a similar explanation for the *first* law of black hole mechanics. The *classical* first law of black hole mechanics looks to me to be just the $\hbar \rightarrow 0$ limit of the first law of thermodynamics.

Carlo: Mmm. . . I see nothing mysterious in the fact that dynamical equations can be obtained as limits of quantum or statistical equations. This is just a minimal consistency requirement.

Don, you are giving me your speculations on how you think Nature works. I listen with interest, but I see no relation with the picture I have given. I do not know if the gravitational field is a fundamental field or not. What I know is that there are some degrees of freedom that in some approximation are well described by the Einstein dynamics and, under the hypotheses of the picture I am describing, that their statistical mechanics describes black hole entropy. The rest is your speculations, not mine!

Don: But what is then your point of view? You have only mentioned the classical theory so far. I get the feeling that you expect the quantum dynamical laws to be much like their classical counterparts...

Carlo: I do. In the sense in which a quantum harmonic oscillator is “much like” a classical oscillator.

Don: ...and therefore that this “quantum Einstein equation” should lead to a “quantum first law of black hole *mechanics*” through an argument

similar to that used to derive the classical first law from the classical equations of motion. . .

Carlo: I am not sure, but I do not expect quantum corrections to be much relevant in this . . .

Don: . . . So, at the quantum level you would 1) find some way to count states to make $A/4G$ into an entropy and then 2) perform some quantum form of Hawking’s calculation to show that a quantum surface gravity κ is associated with radiation at a temperature $T = \hbar\kappa/2\pi$. . .

Carlo: As you know, states of a surface with given area are counted in loop gravity. The key point that makes the picture I am describing meaningful is that the number of “shapes” with a given area is infinite in classical theory, but it turns out to be finite and proportional to the (exponential of the) area in quantum gravity. But let’s not discuss this here. Calculations to derive Hawking’s radiation are in course, but I am not up to date.

Don: . . . this would then turn your quantum first law of black hole mechanics into the (quantum) first law of thermodynamics for black holes. Is this indeed your opinion? If so, do you expect that the 2nd law can be derived the same way?

Carlo: I prefer not to be drawn into speculations about the full quantum statistical mechanics of gravity. In order to understand thermal quantum gravity we have to use a fully background independent framework. To formulate *classical* statistical mechanics in a background independent language is hard enough. Background independent *quantum* statistical mechanics hasn’t been much explored yet, as far as I know. I am convinced that this is where the interesting problems are, and what we should do. My attempt is a first step in this direction, trying to develop steps of classical background independent statistical reasoning. . . For the full quantum theory, I wait for some truly brilliant young kid to appear and do it! For the moment, we do not even know the basic grammar of this. We keep thinking in terms of stuff over a background: I am convinced that this is wrong.

Don: It may well be true we need some conceptual shift away from working around a background . . . But, nevertheless, I feel that you are avoiding the main question. While I am not as sure as Ted that, say, the gravitational

field must in general be like a hydrodynamic quantity, I do find compelling the idea that the hydrodynamic analogy should apply *at least* to certain properties of black holes.

Carlo: Maybe “analogies” can be found, but my expectation is that it must be possible to understand black hole thermodynamics entirely in terms of the quantum and statistical behavior of the gravitational field, without any *need* of resorting to some ‘hydrodynamical’ approximation. But if your criticisms to the picture I have given is only that it does not make use of these analogies, then I am quite happy!

Don: Well, I trust you haven’t forgotten the other issues that Ted and I have raised ...

Carlo: I haven’t, Don! Don’t worry!

In fact, it has been a wonderful discussion and a lovely dinner ... but now I need to move my legs. How about moving to the lounge for an after-dinner drink?

Scene III: States inside the lounge

The discussants have moved to a candle-lit lounge. Glasses of the local digestif, généri, are poured.

Don: Well, it has been an intense day of discussion! You two are quickly wearing me out... However, there is one more point, Ted, on which I would like to press you.

Suppose we consider a setting where we maintain the black hole at constant size (and prevent evaporation) via a steady influx of radiation. Let’s

consider again the horizon-crossing surfaces in my diagram¹⁴ and note that the part of any such surface that lies outside the black hole forms a Cauchy surface for the exterior region.

In this setting an infinite number of Hawking quanta are eventually emitted. But all of these Hawking quanta can in some sense be traced back to any horizon-crossing Cauchy surface, and in particular to the part of this surface very near the horizon. This is precisely your surface system. Now you claim that the black hole entropy is really just the entanglement entropy of the surface system, so that the surface system has a *finite* entropy $S_{BH} = A/4\ell_p^2$, and thus a finite number of states. How then can this finite number of states be consistent with the radiation of an infinite amount of entropy via Hawking quanta?

Ted: Yikes, you're right! But I propose an escape from this conclusion: the standard field theory account of the Hawking effect is unphysical on this point. If there really is a cutoff of the entanglement entropy, and hence a cutoff of the density of outgoing field modes at the horizon, then the outgoing modes cannot arise from a trans-Planckian reservoir at the horizon, but must rather come from elsewhere. I believe that the source is mode conversion from ingoing to outgoing modes near the horizon, as illustrated in various linear field theory models that have been studied¹⁵, including especially Hawking radiation on a falling lattice.

Don: This sort of mode conversion violates Lorentz invariance, and therefore invalidates one of the assumptions used to deduce the thermal nature of the local Rindler horizon. So if you really take this tack, aren't you now in trouble? Even if I grant you the point we shelved earlier, how do you now argue that the surface system is in equilibrium?

Ted: Indeed the picture of vacuum conversion outside the horizon is quite Lorentz violating, and yet in the free fall frame it looks like the usual vacuum. This is why the studies of the Hawking effect in this context have found that the usual Hawking effect is recovered. Hence I think the thermality, while no longer exact to arbitrarily high frequencies, is correct up to the cutoff.

Don: But since your story is that the entropy is dominated by the degrees of freedom at the cutoff, and it is just there that one might expect thermality

¹⁴Recall that these are the hypersurfaces shown on the right in Figure 1.

¹⁵Ted refers to models reviewed in [9].

to break down, are you not in danger of loosing the canonical ensemble just where you most need it?

Ted: Yes, I am in danger. I need another shot of génépi... Ah, that's better. OK, the question is critical: is the mode conversion vacuum outside the black hole horizon a canonical ensemble all the way up to the cut-off? My guess is that the answer is yes, but I can't assert that with total confidence now. Within the linear field theory models it should be straightforward to investigate the question. I'll do that. Suppose it works out to be appropriately thermal. What will be your response?

Don: (*smiling*) Then I will certainly be impressed, but I will still wait for you to fully derive the first law!

Ted: So, Donny boy, after all this, we haven't convinced you yet, eh? Do you still think that black hole entropy has something—or everything—to do with the states *inside* the horizon?

Don: You mean you're not totally devastated and ready to convert to string theory? (*Don smiles mischievously.*)

Ted: No! In fact I'm more convinced than ever that I'm right!

Carlo: I still see the surface picture as the most convincing one, in spite of what I do not understand. Especially since I see no truly reasonable alternative. Don, show your cards: what is your view?

Don: Well, I have to admit that I am attracted to your approach. In particular, there is a result from the paper with Minic and Ross that we've already mentioned which has been on my mind of late. There we found that two observers, one who falls into a black hole and one who remains outside, do not in general agree on the amount of entropy that a given object carries with it into the black hole... This does seem to be suggestive that perhaps the entropy of the black hole itself is somehow “different” as measured by observers inside and outside... but, unfortunately, I don't see precisely where to go from here, and I keep getting hung up on the first law. So, if forced to choose, at the moment I think the more likely scenario is one in which the Bekenstein-Hawking entropy counts the *total* number of black hole microstates. At least, this is what I will take as my working hypothesis.

Ted: Aha! Then now it's our turn to put you on the spot. Can *you* give a statistical derivation of the first law of black hole thermodynamics based on the idea that black hole entropy counts the states on the *inside*?

Don: This, my friends, is child's play... The point is that, if I assume that the *entire* black hole is described by a standard quantum mechanical system, then I can use precisely the usual stat mech argument to which Carlo referred earlier. I place my system in contact with some other stat mech system and let them equilibrate. This means that they randomly exchange energy back and forth.

Now, the most likely macrostate for the joint system is the one that maximizes the total number of microstates available. This occurs when the transfer of a bit of energy dE from one system to the other leaves the total number of states unchanged. Thus, $dS/dE = 1/T$ must be the same for each system, and the first law ($dE = TdS$) is the immediate result. Here, I have merely given a quick summary of the usual story from statistical mechanics... Of course, I need all of the usual caveats about ergodicity, and so forth.

However, before you both assault me physically, let me quickly admit that there are *other* questions that are difficult in this approach. Notably, there is the point we have already visited several times... namely, that the classical physics of the black hole interior does not in an obvious sense *look* like it is approaching equilibrium. Here I can only suggest that this equilibration has something to do with the singularity. To make this fly, I probably need to mutter something about dare I say it?... non-local quantum gravity effects...

Ted and Carlo in unison: You can't be serious!

Ted: Why should causality fail so catastrophically inside a black hole—even one that is a billion kilometers around the horizon?! Curvatures are small and semi-classical gravity should apply deep inside. Whatever happened to the equivalence principle??!

Carlo: Yes ! Quantum gravity effects can very well play a role, but at short scales around the horizon, not hugely non-local effects connecting the singularity with the outside!

Don: Hmmm...., I think I also need another shot of génépi....

I honestly don't know the answer to your question, but progress in string theory over the past decade seems to indicate that this picture is somehow consistent. Here to some extent I refer to the stringy counting of BPS black hole entropy in terms of states on D-branes, but what I really have in mind is Maldacena's AdS/CFT conjecture.

Carlo: Don, my dear friend, this is no fair! I haven't mumbled stuff like "...but progress in loop quantum gravity over the past decade seems to indicate that my picture is consistent". This an entirely different story! If you have a credible story, even incomplete, spit it out! Otherwise, let's stay out from the fog and from belief !

Ted: Uh, oh. Here we go again. Don, you know my arguments¹⁶ that AdS/CFT does not directly give us an answer to questions about black hole entropy ...

Don: Yes, yes, but all I want to take from AdS/CFT is that string theory in AdS space *does* seem to have something fundamentally non-local about it, and that in some sense this non-locality reaches out on a large scale all of the way to the boundary of AdS space! Yet, the theory appears to be consistent and, at least in a classical limit, to provide local, causal, bulk dynamics. Thus, it seems plausible that a similar (though equally unknown) mechanism could be at work in black holes.

I suppose the real point is that I take AdS/CFT as an indication that spacetime itself will not be well-defined at a fundamental level. As a result, I expect that causality will also fail to be precisely well-defined, and thus that some apparently acausal behavior may be allowed. Actually, this seems like a natural consequence of *any* theory of quantum gravity (like, say, one of the standard versions of loop quantum gravity) which does not explicitly build in a causal structure.

Carlo: That spacetime itself is not well-defined at a fundamental level is the working hypothesis of half a century of research in nonperturbative quantum gravity, Don. You know this very well, of course! Loop quantum gravity is just the attempt to make this old idea sufficiently precise to compute with it. Our issue here is whether—if you have a macroscopic black hole—whether acausal behavior can happen macroscopically.

¹⁶Ted refers to his comments in [10].

Ted: Yes, Don, while we don't expect classical causality *per se* in quantum gravity, that does not give us license to commit rampant violations of causality in an otherwise classical setting.

I think there are two levels on which your scenario requires causality violations. One relates to correlations in Hawking radiation. If, as you believe, the entropy *inside* a black hole is $A/4$, then a black hole maintained at constant mass by an influx of energy in a pure state must emit Hawking radiation in a pure state rather than the mixed state predicted by semiclassical analysis. Since this semiclassical prediction is based on analysis at scales very large compared to the Planck length I don't think that quantum gravity effects can invalidate it.

Don: Well, while I have no idea how the details would work, this seems quite plausible to me. After all, as has often been emphasized, one only needs correlations between any two outgoing Hawking particles that are exponentially small in the size of the black hole ... I'm referring to the fact that the non-zero entries in a thermal density matrix are of order $\exp(-S)$, so modifications of order $\exp(-S)$ could suffice to render the density matrix pure ... I don't see how one can rule this out without a complete theory of quantum gravity.

Ted: That seems quite *implausible* to me. The entanglement of each Hawking particle with a partner inside the horizon is inferred from standard QFT vacuum structure at any length scale much smaller than the black hole size, so is immune to quantum gravity effects. Each such pair contributes of order unit entropy, and I don't see how quantum gravity effects could imprint the necessary correlations between the M^2 quanta that are born one by one over a time of order M^3 and propagate separately, far from the black hole out to the hinterlands. I'd be very interested to see you fill in some details, even for a model.

Don: Yes, indeed, this would be a good project ...

Ted: However, there is another level on which your scenario requires *far more* divine intervention. To support your view that equilibrium is maintained inside a black hole, you require not just a subtle quantum gravity effect, but rather total breakdown of the causal *and* dynamical structure of spacetime. You must replace the violent collapse to a spacelike singularity—

which is spread over eons of spacelike proper distance—by some *equilibrium* dynamics!

Don: Sounds like a tall order, doesn't it? Well, perhaps by fleshing out the “surface system” point of view the two of you can save me from such heresy... (*Smiles*)

Of course, you have stated the main problem with this point of view. I really haven't thought enough about this point, but I find myself wondering if it could be resolved by some appropriate “coarse graining” ... Perhaps the non-equilibrium dynamics inside the black hole is just a distraction, while “most” of the black hole degrees of freedom equilibrate? Susskind's ideas on horizon complementarity¹⁷ would fall into this category, though I might hope for something a bit more conventional ... Hmm... It seems like I would need to identify these degrees of freedom to pursue this line of investigation. Unfortunately, I have no concrete suggestions at this time.

Ted: (*with a big grin*) May I suggest, perhaps, the degrees of freedom just outside the horizon?

Don: Your *suggestion* is duly noted ... as I said earlier, I would be only too happy to see you fill in the relevant details ... (*Smiles.*)

My goodness, we've covered a lot of ground today... I wonder if we should try to sum up where our discussion has left us? It seems to me that we all have a lot of work to do to before our pictures of black holes reach the level that we would all truly like to see...

If I've got this right, then for Ted, it's a matter of finding a microscopic explanation—perhaps via the “exterior second law”—for why his surface system “keeps” the entropy it gains from each bit of energy that flows through.

Ted: Ah, the second law, yes... maybe Rafael Sorkin's approach¹⁸ can be made to work... And there's that other question: must I *really* invoke mode conversion to keep the entanglement entropy finite, and if so can the canonical ensemble for the surface system still be justified??

Don: For Carlo, it remains to connect his mixed classical and semi-classical statistical mechanics picture of an ensemble with fixed area to a more complete quantum treatment.

¹⁷Don refers to e.g. [11].

¹⁸Ted refers here to [1, 6].

Carlo: I am happy with all this. There is still confusion under the sky: the situation is excellent. We are still far from the end of physics.

Don: Agreed! And finally, since I advocated treating the Bekenstein-Hawking entropy as counting the number of states in the interior of the black hole, I need some quantum mechanism, perhaps violating our classical notions of locality and causality, to explain how the black interior can be in equilibrium and how the Hawking radiation can be in a pure state...

Ah, it's good to see that we can have a rational discussion about such things... though I am sure that the génépi was essential... (*Don smiles, Ted and Carlo laugh.*) So, my friends, shall we call it a night? I am sure we all look forward to our next discussion.... another place, another time....

And so, our tired and by now slightly inebriated discussants stumble out of the lounge and through the narrow medieval streets of the village under the twinkling stars of the Provencal summer night sky. Each, of course, thinking that they have had the best of the discussion.... but each also having felt more sharply the shortcomings of their own point of view.

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